‡2 Mediæval Indians and the Planets

by Hugh Thurston[1]

A The Kalpa Periods

A1 The main parameter in the mediæval Indian theory of the motion of the planets is the sidereal period. (Yes — even for the inner planets.) They presented this by listing the number of revolutions in a given period. One particularly interesting list is found in the *Brahmasphuta* astronomical tables, compiled in the seventh century A.D. The numbers of revolutions in a period known as a *kalpa* are as follows.

Sun	$4,320,000,000$
Saturn	$146,567,298$
Jupiter	$364,226,455$
Mars	$2,296,828,522$
Venus	$7,022,389,492$
Mercury	$17,936,998,984$

How can we explain these extraordinarily precise figures?

A2 The first few digits in each of the figures cited can be deduced from any reasonably accurate observations. The last few cannot. Let us look at the last four digits of each of the figures.

Saturn	7298
Jupiter	6455
Mars	8522
Venus	9492
Mercury	8984

It would strike a cryptographer at once that these are not random; there is some underlying system. We see this most clearly if we subtract the figures from 10,000. We get:

2702	(a)
3545	(b)
1478	(c)
0508	(d)
1016	(e)

Clearly $e = 2d$. No other regularities are quite as obvious as this, but let us look at the differences; $c - d = 970$, which, subtracted from 10,000 is 9030. This is 10×903, and b is the last four digits of 15×903.

[1] Professor Emeritus of Mathematics, University of British Columbia.

A3 So the number 903 makes itself felt. How can we explain it? Perhaps we have multiples of 903 here. If 2702 is a multiple of 903, the multiplier must end in 4. If we try 4, 14, 24 etc. times 903 we soon find that $34 \times 903 = 30702$ with the same last three digits as a. We can do the same for the other four figures.

$$\begin{aligned} 34 \times 903 &= 30702 \quad (\text{compare } a) \\ 15 \times 903 &= 13545 \quad (\text{compare } b) \\ 26 \times 903 &= 23478 \quad (\text{compare } c) \\ 36 \times 903 &= 32508 \quad (\text{compare } d) \\ 72 \times 903 &= 65016 \quad (\text{compare } e) \end{aligned}$$

So let us add these multiples of 903 to the figures in the *Brahmasphuta*. We get the following results.

$146,598,000$	(multiplier 34)
$364,240,000$	(multiplier 15)
$2,296,852,000$	(multiplier 26)
$7,022,422,000$	(multiplier 36)
$17,937,064,000$	(multiplier 72)

A4 In each case the last digit before the final three zeros is twice the last digit of the multiplier. This is no coincidence: we have not yet found the complete system. If we subtract 2000 times the multipliers from the figures we reduce these digits to zero. Subtracting 2000 is equivalent to adding 8000, so clearly we should have added multiples of 8903, not 903. (But see §A8). Let us do this.

$$\begin{aligned} 146,567,298 &+ 34 \times 8903 &= 146,870,000 \\ 364,226,455 &+ 15 \times 8903 &= 364,360,000 \\ 2,296,828,522 &+ 26 \times 8903 &= 2,297,060,000 \\ 7,022,389,492 &+ 36 \times 8903 &= 7,022,710,000 \\ 17,936,998,984 &+ 72 \times 8903 &= 17,937,640,000 \end{aligned}$$

This suggests that the Indians found approximate figures for the numbers of revolutions in a *kalpa* and subtracted multiples of 8903 to give the final ultra-precise figures. To see why they did this we have to turn from cryptography to astronomy.

A5 One ten-thousandth of a *kalpa* is a *kaliyuga*. The current *kaliyuga* started 4567 *kaliyugas* after the start of the current *kalpa*. There is a remarkable connection between the number 8903 deduced by pure arithmetic and the attested number 4567:

$$4567 \times 8903 = 40660001$$

The approximate numbers of revolutions in a *kalpa* listed above end in four zeros, giving a whole number of revolutions in a *kaliyuga*. Subtracting $8903n$ revolutions in a *kalpa* subtracts $4066.0001n$ revolutions in 4567 *kaliyugas*, i.e. a whole number plus $0.0001n$. A *kalpa* is a period at whose beginning and end the planets were believed to have mean longitude zero. So the *Brahmasphuta's* figures imply that at the beginning of the current *kaliyuga* the mean longitudes of the planets fell short of zero by 0.0034, 0.0015, 0.0026, 0.0036 and 0.0072 revolutions respectively. (0.0015 revolutions is just over half a degree.)

A6 All this suggests that the Indians had figures for the mean longitudes of the planets at the start of the current *kaliyuga* (which was actually -3101/02/18 Julian) and modified the approximate numbers of revolutions to give these figures. To do this, having chosen the number 4567 (no-one seems to know why they chose this particular number), they had only to find a number which, when multiplied by 4567, yields a number ending in 0001. So they had to find whole numbers that satisfy the equation $4567x = 1000y + 1$. This was a standard problem in mediæval Indian mathematics. They used a technique, usually translated as "pulverizer". It is a development of Euclid's algorithm.

A7 There is, however, a much simpler way to find a number x such that $4567 \times x$ ends in 0001. Only the last four digits of x matter. Let us call them $abcd$. The last digit of $4567 \times abcd$ is the last digit of $7 \times d$. For this to be 1, d must be 3. (Proof: multiply 7 successively by 0, 1, 2 ... 9. Only 3 yields a product ending in 1.) The last two digits of $4567 \times abc3$ are the last two digits of $67 \times c3$. For these to be 01, c must be zero. (Proof: multiply 67 by 03, 13, 23 ... 93. Only 03 yields a product ending in 01.) The last three digits of $4567 \times ab03$ are the last three digits of $567 \times b03$. For these to be 001, b must be 9. (Multiply 567 by 003, 103 etc.) Finally, multiplying 4567 by 0903, 1903, etc. we find that only 8903 yields a product ending in 0001.

A8 Instead of adding multiples of 8903 we could subtract multiples of 1097. Because 8903 + 1097 = 10000 this will have the same effect on the longitudes. If we do this we get, instead of the round numbers reconstructed above (§A4), the following round numbers:

$146,530,000$	Saturn
$364,210,000$	Jupiter
$2,296,800,000$	Mars
$7,021,350,000$	Venus
$17,936,920,000$	Mercury

A9 The upshot of all this is that anyone trying to find the source of the figures in the Brahmasphuta is not faced with the gargantuan task of explaining the extraordinarily precise figures first quoted but with the much less daunting task of explaining the round numbers reconstructed here.

B Reference

Brāhma-sphuta Siddhānta, Indian Institute of Astronomical and Sanskrit research, New Delhi, 1966. (This edition has a long and useful introduction in English.)