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In the 2002 March *Isis* (HistSciSoc), Harvard's O.Gingerich tries countering H.Thurston by ***defending fraudulent "observations"*** (Ptolemy's Venus data, incl. 2 different dates for the same event!), claiming the math was so hard that Ptolemy was ***forced to hoax***. The present *DIO* is comprised of demonstrations that a problem which stumped Gingerich's Greatest ancient astronomer is easily solved: even by a highschooler, as the below program proves. [Thurston's geocentric method (‡7 §E) finds identical orbital elements even more easily.]

The "Essentially Insoluble" * — Solved by 10th Grade Math

```
10 Q$="Q":IF Q$<>"C"THEN F=2 ELSE F=1:REM"Q"equant,"C"eccentric
20 F$="e=###.###          A=####.#":IF Q$<>"C"THEN PRINT"Eqt"ELSE PRINT"Ecc"
30 S=3:E$="n=##          ":G$=F$+"          r=###.###":H$=E$+G$
40 PI=3.141593:C=180/PI:C$=E$+F$:IF Q$="C"GOTO 70
50 E0=1/60:A=60:X0=E0*COS(A/C):Y0=E0*SIN(A/C)
60 PRINT USING C$;N,E0*60,A
70 G(1)=+(43+24/60):V(1)=148+34/60:REM AD138Evening
80 G(2)=+(48+17/60):V(2)=015+08/60:REM AD140Evening
90 G(3)=-(47+09/60):V(3)=064+06/60:REM AD140Morning
100 FOR I=1 TO 3
110 G(I)=G(I)/C:V(I)=V(I)/C:L(I)=V(I)-G(I)
120 L=L(I)*C:L=L-360*INT(L/360):L(I)=L/C
130 NEXT I
140 FOR I=1 TO 3:G=L(I)-A/C:IF Q$="C"GOTO 160
150 H=E0*SIN(G):G=G-ATN(H/SQR(1-H^2))
160 QX=COS(G)+E0:QY=SIN(G):GQ=C*ATN(QY/QX):IF QX<0 THEN GQ=GQ+180
170 R=SQR(QX^2+QY^2):LQ=GQ+A
180 X(I)=R*COS(LQ/C):Y(I)=R*SIN(LQ/C)
190 M(I)=TAN(V(I)):B(I)=Y(I)-M(I)*X(I):NEXT I
200 FOR I=1 TO 3
210 J=I-3*INT(I/3)+1
220 IF SGN(G(I))=SGN(G(J)) THEN SH=PI ELSE SH=0
230 P(I)=-(B(I)-B(J))/(M(I)-M(J)):Q(I)=(M(I)*B(J)-M(J)*B(I))/(M(I)-M(J))
240 U(I)=(V(I)+V(J)-SH)/2:H(I)=(V(I)-V(J)-SH)/2
250 T(I)=TAN(U(I)):F(I)=Q(I)-T(I)*P(I):NEXT I
260 FOR I=1 TO 3:J=I-3*INT(I/3)+1
270 C(I)=-(F(I)-F(J))/(T(I)-T(J))
280 D(I)=T(I)*C(I)+F(I)
290 S(I)=SQR((P(I)-C(I))^2+(Q(I)-D(I))^2)
300 R(I)=ABS(S(I)*SIN(H(I))):NEXT I
310 X0=X0-C(3)/F:Y0=Y0-D(3)/F
320 R0=R(3)*60:N=N+1
330 E0=SQR(X0^2+Y0^2):E=60*E0:A=C*ATN(Y0/X0):IF X0<0 THEN A=A+180
340 IF Q$<>"C"THEN PRINT USING H$;N,E,A,R0 ELSE PRINT USING G$;E,A,R0
350 IF N<S AND Q$<>"C" GOTO 140
```

* See ‡6 §B4. The above BASIC program performs the method developed within from ‡6 §D (teen-age-level ruler&compass crux) through to §G (iterative application to equant planet model).

‡7 Unveiling Venus

by Hugh Thurston[1]

Although we may never know for certain, it is interesting to speculate how early people, the Greeks in particular, came to deal with the motion of Venus, their extant efforts culminating in the theory described in Ptolemy's *Syntaxis*.

A Early Days

A1 Early people will certainly have seen a bright planet (which the Greeks called *Eosphoros*) visible in the morning for several months at a time, and a bright planet (*Phosphoros*) visible in the evening, but not during the time when *Eosphoros* was visible. Quite early, before the time of Eudoxus, the Greeks realized that the two were in fact one planet (our Venus). The Babylonians, the Chinese, and even the relatively unsophisticated Mayas, also realized this.

A2 Also quite obvious were the retrogressions of the three visible outer planets. Early on it became clear that their motion was to-and-fro round a steadily moving "mean planet". (Literally so called by the Hindus. The Sanskrit is *madhya graha*.) Eudoxus had the planet moving in a figure-of-eight; and the Chinese had it moving in the shape of a willow-leaf. A good quantitative theory became possible when some genius thought of having it move round on an epicycle.

A3 The change from evening star to morning star is a retrogression (past the sun) so the motion of Venus can also be dealt with by using an epicycle. The center of Venus' epicycle keeps in line with the mean Sun.

B Epicycles and Eccentrics

B1 Ptolemy used the mean longitude, not the true longitude, of the sun, and earlier astronomers may well have done the same: the doctrine of regular circular motion, which probably underlay this choice, is older than Ptolemy.

B2 If the speed of the planet round C, the speed of C round the earth, the radius of the epicycle and the distance of C from the earth were all constant, retrogressions would all be equally long. They are not, so something has to change.

B3 I suggest that the first modification to the theory was to make the centre O of the orbit of C a point distinct from the earth T, used by Hipparchus for the sun with considerable success.

B4 We need to find the direction of O from T, the distance e of O from T, the radius r of the epicycle and the radius R of the orbit of C. The ancients could not measure astronomical distances, so they could find only the mutual ratios of e, r, and R.

B5 The astronomers started with a list of timed longitudes of Venus. For each they could calculate the mean longitude of the Sun, MLS. Its difference from the longitude of Venus is the mean elongation. They worked with the maximum mean elongation: MME.

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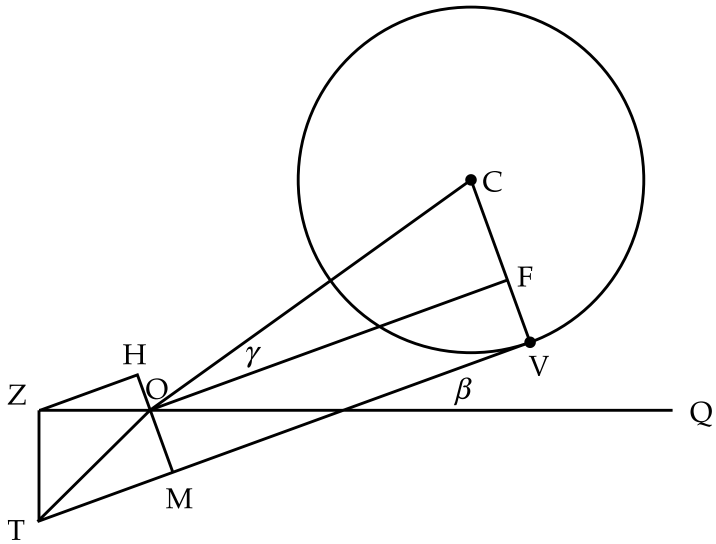

Figure 1: See §D2.

B6 There are two things that this could mean. If, starting just before the middle of a period of visibility, they measured the longitude of Venus each night and picked the greatest of the mean elongations, that would give a maximum with respect to time.

B7 Alternatively, if they had a list of mean elongations they could look at all the mean elongations for a given MLS and pick the greatest. This gives the greatest ME for a given position of the epicycle. Clearly, when Venus is at the greatest ME for a given position of the epicycle, the line of sight from the earth to Venus is an exact[2] tangent to the epicycle. From now on, this version is what I mean by MME. It is the version used by Dennis Duke in ‡5 of this issue of *DIO* and is the version that appears in Ptolemy's calculations. For each MLS there will be two MMEs, one to the east and one to the west.[3] Their sum gives the apparent size of the epicycle. The first step is to find the longitude of O. There are at least three ways to do this.

B8 Method 1: Find the position of C for which the apparent size of the epicycle is greatest and the position for which it is least. To do this the astronomer would tabulate the MMEs in the order of their MLSs, for each western MME find by interpolation the eastern MME for the same MLS, add the two MMEs together, note how the sum varies, and find by interpolation the MLSs giving the greatest and least values. (If they do not differ by 180°, there is a problem.)

[2] The fact that the non-tangency rule noted at ‡6 §F applies to greatest MEs with respect to time but not to those with respect to MLS was first mentioned in 1977 by F.W.Sawyer (ΠΡΙΣΜΑΤΑ, pages 169 to 181).

[3] The difference between the two kinds of MME can be seen beautifully clearly in Fig.1 of Dennis Duke's article [‡5] (Eastern elongations are seen in the evening). The dotted lines are graphs of ME against MLS. MLS increases steadily with time, so the highest point on each gives a maximum with respect to time. The points on the outer envelope give the greatest ME for each position of the epicycle.

B9 Method 2: Find an eastern MME equal to a western MME. A moderately large list of MMEs would give a reasonable chance of finding such a pair. Ptolemy said that he used this method, and, assuming that the line OT runs halfway between the two equal MLSs, found that it extends along the diameter 55°-to-235°. Comparing the apparent size of the epicycle when the MLS is near to 55° with the size when it is near to 235° showed that the longitude of O is 55°.

B10 The trouble here is that the equality of the two MMEs does not guarantee that OT runs halfway between the MLSs. This is easily seen from Dennis Duke's graph [1] of eastern MME against MLS [Figure 1 of article ‡5 in this *DIO*].

B11 For each eastern MME (other than the greatest and least ones) there are two MLSs. If comparing one with an equal western MME (as graphed at ‡5 Figure 2) gives the right direction to OT, comparing the other one with the same western MME will not. Ptolemy's result is roughly correct, so if he actually used this method he must have been lucky in picking a well-matched pair.

B12 Method 3: Use three MMEs. The appendix (§D) shows how.

B13 Admittedly, we have no direct evidence that any ancient astronomer used method 3, but perhaps we should not expect any.

B14 The next step is to find e and r in terms of R. If we have used method 3 there is no problem. If we have used method 1 or 2 the most straightforward way is to use two MMEs when C is on the line OT. Ptolemy claimed to have used this method and explained it in chapter 2 of book 10 of the *Syntaxis*.

B15 The trouble here lies in finding MMEs when C is in the right position. Suppose that when C is on the line, say at 55°, Venus is at the point V of the epicycle. C will reach 55° at yearly intervals. The synodic period of Venus is very close to 8/5 of a year. Let V, W, X, Y, and Z divide the epicycle into five equal arcs. The next five times that C is at 55° Venus will be at Y, W, Z, X, and back at V; not exactly but close. The positions of Venus when C is at 55° will be close to these five points for a long time. Unless we are lucky and one of them yields a MME we will not find what we want. Ptolemy said that he found it (on May 20th, 129 AD).

C The Equant

C1 Ptolemy's final theory used an equant. Dennis Duke has shown [‡5 §§E1-E4] how Ptolemy might have been led to this. Duke's reasoning can easily be restated in ancient Greek fashion. Duke also explained [‡5 §C2] how the ancients might have found the radius of the epicycle.

C2 Several writers have suggested that Ptolemy adjusted (or even fabricated) his observations. In particular, Swerdlow wrote that Ptolemy "could not have observed some of the reported elongations" (he certainly could not have observed two MMEs 37 days apart) and "Ptolemy's adjustments of whatever he observed were of the order of 1°" [2, page 54].

D Appendix

D1 Figure 1 shows an occasion when the sight-line from the earth T to Venus V is a tangent to the epicycle. OQ points in the direction of zero longitude. Angle COQ is the mean longitude of the sun. The angle between TV and OQ is the longitude β of Venus. CVT is a right angle.

D2 Drop perpendiculars OF to CV, OM to TV, TZ to OQ, and ZH to OM. Angle COF is the elongation γ. (OT is the eccentricity, and TO points towards the apogee.)
Angle ZTS is $90° - \beta$. Then HS = (ZT/120)·chd($180° - 2\beta$). Angle HZO = β. Then HO = (OZ/120)·chd(2β). Thus, r = CV = CF + FV = CF + OM = CF + HM − HO = (1/2)·chd(2γ) + (ZT/120)·chd($180° - 2\beta$) − (OZ/120)·chd(2β).

D3 Given three such occasions, we have three expressions for r, giving [by equating the three expressions] two linear equations in the two unknowns ZT and OZ, which the Greeks could solve. That gives both the distance OT [eccentricity e] and the angle between OZ and OT and hence the direction [apogee A] of O from T. Then any one of the three expressions gives r. This method has been applied to real data by Dennis Rawlins [3] using modern mathematics (coordinate geometry) and by Dennis Duke, using vectors (unpublished).

References

[1] Fax [‡5 Figure 1] on July 3rd, 2002, from Dennis Duke, Florida State University.
[2] Noel M. Swerdlow. Ptolemy's theory of the inferior planets, *Journal for the History of Astronomy* volume 20 (1989) pages 29 to 60.
[3] Dennis Rawlins, The Crucial-Test, *DIO* volume 11 pages 54 and 70 to 90 [paper ‡6: that preceding the present one].

E Program for Thurston Geocentric Method [2003 DR Addendum]

Hugh Thurston's geocentric method[4] can be programmed just as readily as DR's heliocentric method (p.54). To save space — and highlight differences in the two approaches — we below provide merely a dozen lines of the geocentric program, since all the other lines are identical to those of p.54's program. Thus, a simple 12-line replacement-operation (merge-command) upon the 35-line heliocentric program on p.54 produces the full 35-line geocentric program. Despite the methods' differing math steps and reference-frames, the results of the two programs are the same, both for the equant[5] (where every loop's results are absolutely identical: e.g., ‡6 §G9) and the eccentric model.

```
130 N(I)=1:NEXT I
150 H=E0*SIN(G):G0=G:G=G−ATN(H/SQR(1−H^2)):N(I)=SIN(G)/SIN(G0)
190 NEXT I
220 IF SGN(G(I))=SGN(G(J)) THEN SI=1 ELSE SI=−1
230 J(I)=COS(V(J))−SI*COS(V(I))
240 K(I)=−(SIN(V(J))−SI*SIN(V(I)))
250 Z(I)=SIN(G(J))*N(J)−SI*SIN(G(I))*N(I):NEXT I
270 W(I)=J(J)*K(I)−J(I)*K(J)
280 A(I)=(J(J)*Z(I)−J(I)*Z(J))/W(I)
290 O(I)=(Z(J)*K(I)−Z(I)*K(J))/W(I)
300 R(I)=ABS(N(I)*SIN(G(I))−O(I)*COS(V(I))+A(I)*SIN(V(I))):NEXT I
310 X0=A(3)/F:Y0=O(3)/F
```

[4] The Thurston method is mathematically quicker than the DR method (‡6 or p.54); though the latter solution, if performed graphically, could hardly be more elementary.

[5] Adjusting the Thurston method for the equant model (to produce the present versatile BASIC program) requires only that in Fig.1, the epicycle-center (still moving at constant angular velocity around O, now the equant point) is at unit distance from OT's midpoint (instead of from O), which will be e (instead of $e/2$) distant from O and from T. Thus (using the law of sines & ‡6 eq.33), OC equals sin$u$/sin$g$ (instead of unity). The appropriate adjustments (to equant model) of the §D3 method are built into lines 310(&10), 250&300(&150) of the geocentric program.

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